Archimedes' principle

It is named after Archimedes of Syracuse, who first discovered this law. According to Archimedes' principle, "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Vitruvius (De architectura IX.9–12) recounts the famous story of Archimedes making this discovery while in the bath. He was given the task of finding out if a goldsmith, who worked for the king, was carefully replacing the king's gold with silver. While doing this Archimedes decided he should take a break so went to take a bath. While entering the bath he noticed that when he placed his legs in, water spilled over the edge. Struck by a moment of realization, he shouted "Eureka!" He informed the king that there was a way to positively tell if the smith was cheating him. Knowing that gold has a higher density than silver, he placed the king's crown and then a gold crown of equal weight into a pool. The king's crown caused more water to overflow, showing that it had a greater volume for the same weight. It was, therefore, less dense than gold, and Archimedes concluded that it contained silver, causing the smith to be executed. The actual record of Archimedes' discoveries appears in his two-volume work, *On Floating Bodies*. The ancient Chinese child prodigy Cao Chong (196–208 AD) also applied the principle of buoyancy in order to accurately weigh an elephant, as described in the Sanguo Zhi, also known as the Records of Three Kingdoms.

Archimedes' principle does not consider the surface tension (capillarity) acting on the body.[1]

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Suppose a rock's weight is measured as 10 newtons when suspended by a string in a vacuum. Suppose that when the rock is lowered by the string into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyant force: 10 − 3 = 7 newtons. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water.

The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{\text{density of object}}{\text{density of fluid}} = \frac{\text{weight}}{\text{weight} - \text{apparent immersed weight}}$$

Forces and equilibrium

This is the equation to calculate the pressure inside a fluid in equilibrium. The corresponding equilibrium equation is:

$$\mathbf{f} + \text{div}\sigma = 0$$

where $\mathbf{f}$ is the force density exerted by some outer field on the fluid, and σ is the stress tensor. We know that in our case the stress tensor is proportional to the identity tensor: $\sigma_{ij} = -p\delta_{ij}$. Here δ_{ij} is the kronecker delta symbol. Using this the above equation becomes:

$$\mathbf{f} = \nabla p$$

Now let's assume that the outer force field is conservative, that is it can be written as the negative gradient of some scalar valued function: :$\mathbf{f} = -\nabla\Phi$. Hence we have:

$$\nabla(p + \Phi) = 0 \Longrightarrow p + \Phi = \text{constant.}$$

As we see, we got that the shape of the open surface of a fluid equals the equipotential plane of the applied outer conservative force field. Now let's put the z axis pointing downwards. In our case we have gravity, so $\Phi = -\rho g z$ where g is the gravitational acceleration, ρ is the mass density of the fluid. Let the constant be zero, that is the pressure zero where z is zero. So the pressure inside the fluid, when it is subject to gravity:

$$p = \rho g z$$

So as we see, pressure increases with depth below the surface of a liquid, as z denotes the distance from the surface of the liquid into it. Any object with a non-zero vertical depth will have different pressures on its top and bottom, with the pressure on the bottom being greater. This difference in pressure causes the upward buoyancy forces.

The buoyant force exerted on a body can now be calculated easily, since we know the internal pressure of the fluid. We know that the force exerted on the body can be calculated by integrating the stress tensor over the surface of the body:

$$\mathbf{F} = \oint \sigma \, d\mathbf{A}$$

The surface integral can be transformed into a volume integral with the help of the Gauss-Ostrogradsky theorem :

$$\mathbf{F} = \int \text{div}\sigma \, dV = -\int \mathbf{f} \, dV = -\rho \mathbf{g} \int dV = -\rho \mathbf{g} V$$

where *V* is obviously the measure of the volume in contact with the fluid, that is the volume of the submerged part of the body. Since the fluid doesn't exert force on the part of the body which is outside of it.

The magnitude of buoyant force may be appreciated a bit more from the following argument. Consider any object of arbitrary shape and volume*V* surrounded by a liquid. The force the liquid exerts on an object within the liquid is equal to the weight of the liquid with a volume equal to that of the object. This force is applied in a direction opposite to gravitational force that is, of magnitude:

$\rho V_{disp} g$, where ρ is the density of the liquid, V_{disp} is the volume of the displaced body of liquid , and g is the gravitational accelerationat the location in question.

Now, if we replace this volume of liquid by a solid body of the exact same shape, the force the liquid exerts on it must be exactly the same as above. In other words the "buoyant force" on a submerged body is directed in the opposite direction to gravity and is equal in magnitude to :

$$\rho V g$$

The net force on the object is thus the sum of the buoyant force and the object's weight

$$F_{\text{net}} = mg - \rho V g$$

If the buoyancy of an (unrestrained and unpowered) object exceeds its weight, it tends to rise. An object whose weight exceeds its buoyancy tends to sink.

Commonly, the object in question is floating in equilibrium and the sum of the forces on the object is zero, therefore;

$$mg = \rho V g$$

and therefore;

$$m = \rho V$$

showing that the depth to which a floating object will sink (its "**buoyancy**") is independent of the variation of the gravitational acceleration at various locations on the surface of the Earth.

(*Note: If the liquid in question is seawater, it will not have the same density (ρ) at every location. For this reason, a ship may display aPlimsoll line.*)

It is common to define a *buoyant mass* m_b that represents the effective mass of the object with respect to gravity

$$m_b = m_{\text{o}} \cdot \left(1 - \frac{\rho_{\text{f}}}{\rho_{\text{o}}}\right)$$

where m_{o} is the true (vacuum) mass of the object, whereas ρ_o and ρ_f are the average densities of the object and the surrounding fluid, respectively. Thus, if the two densities are equal, $\rho_o = \rho_f$, the object appears to be weightless. If the fluid density is greater than the average density of the object, the object floats; if less, the object sinks.

Compressive fluids

The atmosphere's density depends upon altitude. As an airship rises in the atmosphere, its buoyancy decreases as the density of the surrounding air decreases. As a submarine expels water from its buoyancy tanks (by pumping them full of air) it rises because its volume is constant (the volume of water it displaces if it is fully submerged) as its weight is decreased.

Compressible objects

As a floating object rises or falls, the forces external to it change and, as all objects are compressible to some extent or another, so does the object's volume. Buoyancy depends on volume and so an object's buoyancy reduces if it is compressed and increases if it expands.

If an object at equilibrium has a compressibility less than that of the surrounding fluid, the object's equilibrium is stable and it remains at rest. If, however, its compressibility is greater, its equilibrium is then unstable, and it rises and expands on the slightest upward perturbation, or falls and compresses on the slightest downward perturbation.

Submarines rise and dive by filling large tanks with seawater. To dive, the tanks are opened to allow air to exhaust out the top of the tanks, while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has neutral buoyancy and will remain at that depth. Normally, precautions are taken to ensure that no air has been left in the tanks. If air were left in the tanks and the submarine were to descend even slightly, the increased pressure of the water would compress the remaining air in the tanks, reducing its volume. Since buoyancy is a function of volume, this would cause a decrease in buoyancy, and the submarine would continue to descend.

The height of a balloon tends to be stable. As a balloon rises it tends to increase in volume with reducing atmospheric pressure, but the balloon's cargo does not expand. The average density of the balloon decreases less, therefore, than that of the surrounding air. The balloon's buoyancy decreases because the weight of the displaced air is reduced. A rising balloon tends to stop rising. Similarly, a sinking balloon tends to stop sinking.

Density

If the weight of an object is less than the weight of the displaced fluid when fully submerged, then the object has an average density that is less than the fluid and has a buoyancy that is greater than its own weight. If the fluid has a surface, such as water in a lake or the sea, the object will float at a level where it displaces the same weight of fluid as the weight of the object. If the object is immersed in the fluid, such as a submerged submarine or air in a balloon, it will tend to rise. If the object has exactly the same density as the fluid, then its buoyancy equals its weight. It will remain submerged in the fluid, but it will neither sink nor float. An object with a higher average density than the fluid has less buoyancy than weight and it will sink. A ship floats because although it is made of steel, which is much denser than water, it encloses a volume of air which is lighter than water, and the resulting shape has an average density less than that of the water.

Specific gravity is defined as the ratio of the density of a given solid or liquid substance to the density of water at a specific temperature and pressure, typically at 4°C (39°F) and 1 atm (760.00 mmHg) , making it a dimensionless quantity (see below). Substances with a specific gravity greater than one are denser than water, and so (ignoring surface tension effects) will sink in it, and those with a specific gravity of less than one are less dense than water, and so will float in it. Specific gravity is a special case of, or in some usages synonymous with, **relative density**, with the latter term often preferred in modern scientific writing. The use of specific gravity is discouraged in technical use in scientific fields requiring high precision — actual density (in dimensions of mass per unit volume) is preferred.

Specific gravity, *SG*, is expressed mathematically as:

$$\mathrm{SG} = \frac{\rho_{\mathrm{substance}}}{\rho_{\mathrm{H_2O}}}$$

where $\rho_{\mathrm{substance}}$ is the density of the substance, and $\rho_{\mathrm{H_2O}}$ is the density of water. (By convention ρ, the Greek letter rho, denotes density.) The density of water varies with temperature and pressure, and it is usual to refer specific gravity to the density at 4°C (39.2°F) and a normal pressure of 1 atm. The given temperature and pressure are preferred because it is when water has its maximum density. In this case $\rho_{\mathrm{H_2O}}$ is equal to 1000 kg·m^{-3} in SI units (or 62.43 lb$_m$·ft^{-3} in United States customary units).

Given the specific gravity of a substance, its actual density can be calculated by inverting the above formula:

$$\rho_{\mathrm{substance}} = \mathrm{SG} \times \rho_{\mathrm{H_2O}}$$

Occasionally a reference substance other than water is specified (for example, air), in which case specific gravity means density relative to that reference.

Specific gravity is, by definition, dimensionless and therefore not dependent on the system of units used (e.g. slugs·ft^{-3} or kg·m^{-3}). However, the two densities must be converted to the same units before carrying out the numerical ratio calculation.

For information about the measurement of and uses of specific gravity, see relative density.